

ARJUNA

JEE

LECTURE NO. - 5

Basic Math



MSM SIR



TABLE OF CONTENT



✓ **01**

No-Ne Expressions

✓ **02**

Questions Practice



QUADRATIC KA GYAAN

$$ax^2 + bx + c = 0$$

$$D = b^2 - 4ac$$

$$D < 0$$

Then,

' $ax^2 + bx + c$ ' cannot be factorized.

Reason in
Quadratic
chapter



QUADRATIC KA MAHA-GYAAN



$$E = ax^2 + bx + c$$

$$\downarrow x=2; E = a(2)^2 + b(2) + c \\ = (4a + 2b + c) \quad \checkmark$$

$$\nearrow x=0 \quad E = a(0)^2 + b(0) + c = c \quad \checkmark$$



$$E = ax^2 + bx + c$$

$$\wedge \begin{cases} a > 0 \\ D < 0 \end{cases}$$

Then

$$\{E > 0\}$$

~~$E \leq 0$~~

$$\forall x \in \mathbb{R}$$



$$\wedge \begin{cases} a < 0 \\ D < 0 \end{cases}$$

Then

$$E < 0$$

~~$E \geq 0$~~

$$\forall x \in \mathbb{R}$$



Reason
in
Quadratic
chapter

Concept used in
Quadratic chap;
Calculus, Trigon



Q.

For what values of 'k' is the quadratic polynomial

$$((k-2)x^2 + 8x + k + 4)$$

negative for all x belonging to real number.

$$E = (k-2)x^2 + 8x + (k+4)$$

$$E < 0 \quad \forall x \in \mathbb{R}$$

$$\wedge \begin{cases} a = k-2 < 0 \\ D = (8)^2 - 4(k-2)(k+4) < 0 \end{cases}$$

Common 4 $\rightarrow 64 - 4(k^2 + 2k - 8) < 0$

$$16 - (k^2 + 2k - 8) < 0$$

$$24 - k^2 - 2k < 0$$

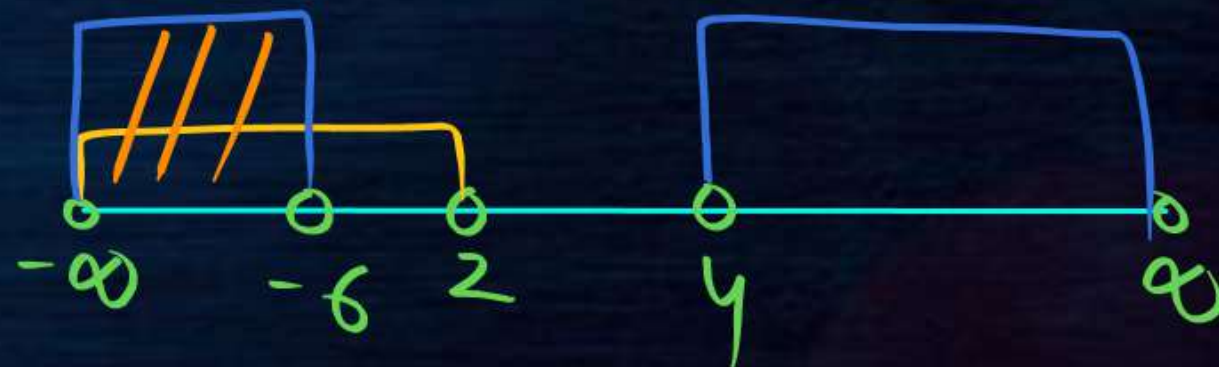
$\times (-1)$

$$k^2 + 2k - 24 > 0$$

$$k-2 < 0 \Rightarrow k \in (-\infty, 2)$$

$$(k+6)(k-4) > 0$$

$$k \in (-\infty, -6) \cup (4, \infty)$$



$$k \in (-\infty, -6) \text{ And } (4, \infty)$$



Q.

For what values of 'k' is the quadratic polynomial

$$((k-2)x^2 + 8x + k + 4)$$

positive for all x belonging to real number.

$$E = (k-2)x^2 + 8x + (k+4)$$

$$E > 0 \quad \forall x \in \mathbb{R}$$

(condⁿ →

$$\textcircled{1} k-2 > 0 \quad \checkmark$$

$$\textcircled{2} 64 - 4(k-2)(k+4) < 0$$

$$4(16 - (k-2)(k+4)) < 0$$

$$16 - (k^2 + 2k - 8) < 0$$

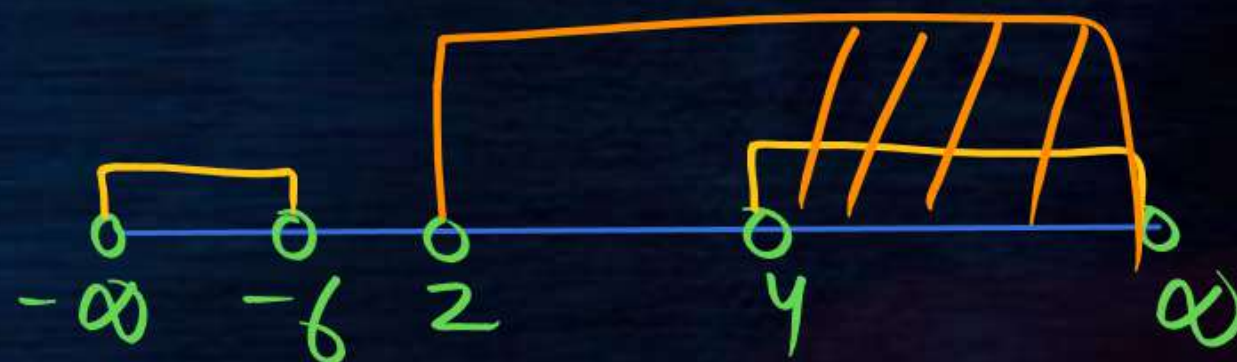
$$\textcircled{1} \quad 24 - k^2 - 2k < 0$$

$$k^2 + 2k - 24 > 0$$

$$\textcircled{1} k \in (2, \infty) \quad \checkmark$$

$$\textcircled{2} (k+6)(k-4) > 0$$

$$k \in (-\infty, -6) \cup (4, \infty) \quad \checkmark$$



$$k \in (4, \infty) \text{ Ans.}$$



Summary

$$E = ax^2 + bx + c$$

① $D < 0 \rightarrow$ cannot be factorized.

$$\wedge \begin{cases} a > 0 \\ D < 0 \end{cases}$$

$$E > 0 \quad \forall x \in \mathbb{R}$$

$$\wedge \begin{cases} a < 0 \\ D < 0 \end{cases}$$

$$E < 0 \quad \forall x \in \mathbb{R}$$

MODULUS KA GYAAN

→ "absolute value Funcⁿ"

" $|x|$ "

$$\textcircled{1} |6| = 6$$

$$\textcircled{2} |-6| = 6$$

$$\textcircled{3} |0| = 0$$

★ New item →

$$|x| = \begin{cases} +x; & x \geq 0 \\ -x; & x \leq 0 \end{cases}$$

ex) →

$$\bullet |6| = +6$$

$$\bullet |-6| = -x = -(-6) = 6$$

$$\bullet |0| = +0 = -0$$



TYPE 1



Q.

Simplify the following:

(i) $|\sqrt{3} - 1|$
(ii) $|2 - \pi|$

(i) $+(\sqrt{3} - 1)$ Ans

(ii) $-(2 - \pi) = (\pi - 2)$ Ans.



Q.

Simplify the following:

~~(i)~~ $\frac{|x|}{x}$, if $x > 0$ $x \neq 0$

~~(ii)~~ $\frac{|x|}{x}$, if $x < 0$

(i) $\frac{+x}{x} = 1$

(ii) $\frac{-x}{x} = -1$



SQUARE ROOT KA GYAAN

By Defⁿ,

$$\sqrt{4} = 2, \quad \cancel{-2}$$



$$\sqrt{x^2} = |x|$$

$$\sqrt{4} \rightarrow \sqrt{(2)^2} = |2| = 2 \checkmark$$

$$\sqrt{4} \rightarrow \sqrt{(-2)^2} = |-2| = -(-2) = 2 \checkmark$$

ex) $\rightarrow \sqrt{9} = 3$ Ans

~~3~~



Q.

Simplify : $\sqrt{16a^4b^6c^3}$, if $b < 0$ and $c > 0$.



?

$$\sqrt{(4a^2b^3c)^2 \cdot c} = \sqrt{(4a^2b^3c)^2} \cdot \sqrt{c}$$

$$= |4a^2b^3c| \cdot \sqrt{c}$$

(P N P)

Negative

$$= (-4a^2b^3c\sqrt{c}) \text{ Ans.}$$



Gyaan →

$$|z| = K \quad ; \quad K \geq 0$$

$$\Rightarrow z = \pm K$$

TYPE 2

Q) → $|x| = 2$; $x = ?$

solⁿ → $x = \pm 2$ Ans

Q → $|x| = -2$, $x = ?$

solⁿ → $x \in \emptyset$



Q.

Find x if

?

$$(i) |x - 2| = 2$$

$$(ii) |x - 4| = -2$$

$$(i) |x - 2| = 2$$

$$\Rightarrow \text{modulus} = x - 2$$

$$\rightarrow x - 2 = \pm 2$$

$$x = 2 \pm 2$$

$$x \in \{0, 4\} \text{ Ans}$$

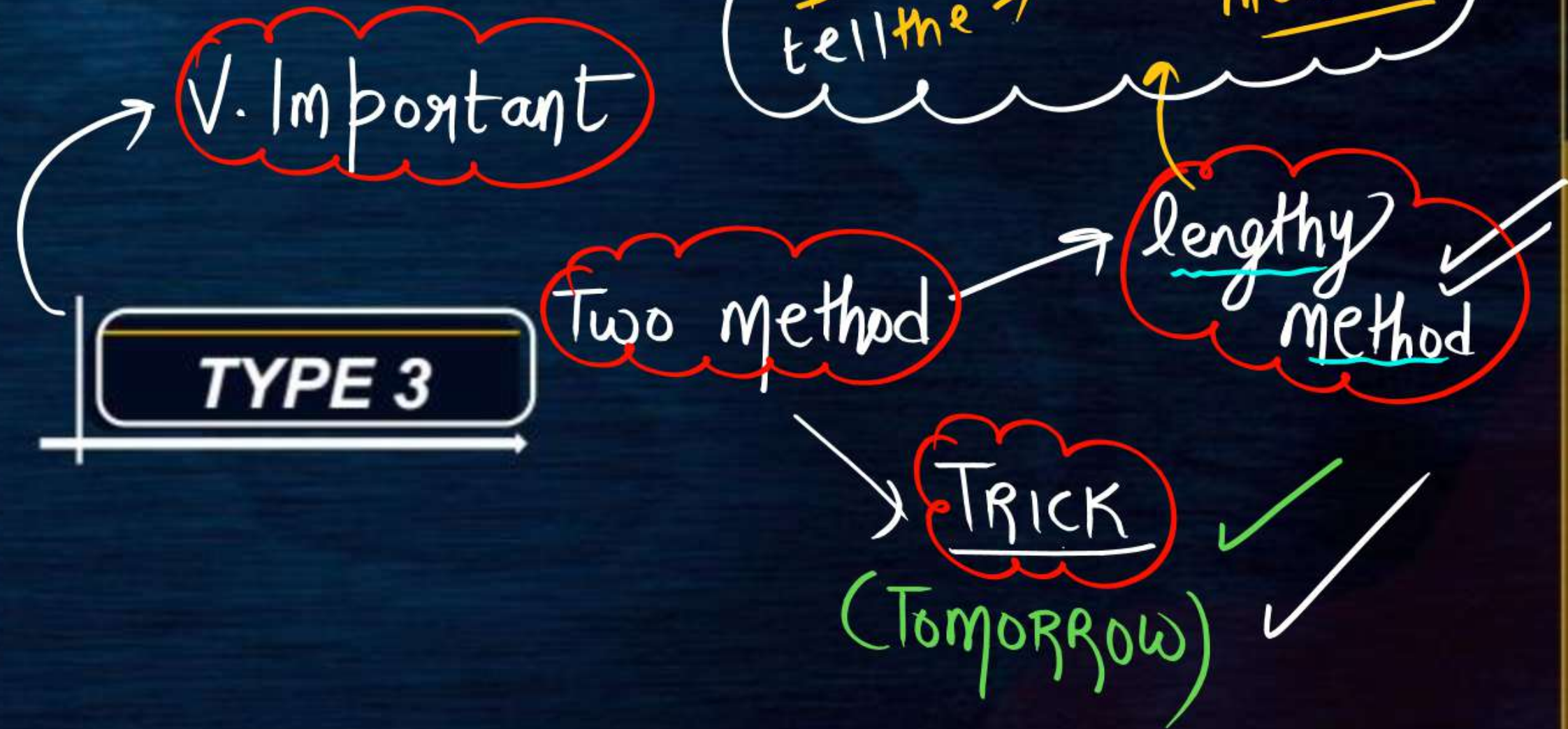
$$(ii) |x - 4| = -2$$

NOT POSSIBLE

$$x \in \phi \text{ Ans}$$



When more than
one mod
is added or
subtracted



Q.

$$|x-2| + |x-4| = 7, \text{ Solve.}$$

?



Case 1 $\Rightarrow x \in [4, \infty)$ ✓

$$+(x-2) + (x-4) = 7$$

$$2x - 6 = 7$$

$x = \frac{13}{2}$ ✓

Case 2 $\Rightarrow x \in [2, 4)$

$$+(x-2) - (x-4) = 7$$

$$x - 2 - x + 4 = 7$$

$$2 = 7 \text{ (Invalid)}$$

$x \in \emptyset$

Case 3 $\Rightarrow x \in (-\infty, 2)$

$$-(x-2) - (x-4) = 7$$

$$-x + 2 - x + 4 = 7$$

$$-2x + 6 = 7$$

✓ $x = -\frac{1}{2}$

final

Ans

$$\Rightarrow x \in \left\{ -\frac{1}{2}, \frac{13}{2} \right\} \text{ Ans}$$

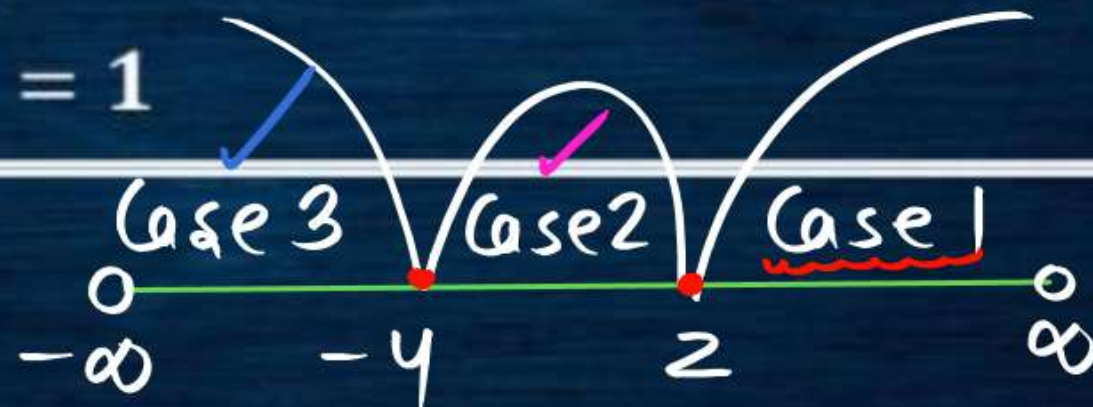


Q.

Solve the equation:-

$$2|x-2| - 3|x+4| = 1$$

final ans $\rightarrow$
 $x \in \{-\frac{9}{5}, -15\}$



Case 1 $\Rightarrow x \in [2, \infty)$

$$2(x-2) - 3(x+4) = 1$$

$$2x - 4 - 3x - 12 = 1$$

$$-x = 17$$

$$x = -17$$

$\downarrow$ Discarded

Case 2 $\Rightarrow x \in [-4, 2)$

$$-2(x-2) - 3(x+4) = 1$$

$$-2x + 4 - 3x - 12 = 1$$

$$-5x - 8 = 1$$

$$x = -\frac{9}{5} = -1.8$$

Case 3 $\Rightarrow x \in (-\infty, -4)$

$$-2(x-2) + 3(x+4) = 1$$

$$-2x + 4 + 3x + 12 = 1$$

$$x = -15$$



ARJUNA

Q. Simplify $f(x) = |x - 1| - 3|2 - x| + 5|x - 7|$

"Challenge HW"

"JBK Done"

?



"ZARFA DHYAN SE"

TYPE 4



Q.

$$||x| - 1| - 2 = 3 \text{ find } x.$$

"E"

✓ $|E| = 3$

✓ $E = \pm 3$

$$(|x| - 1) - 2 = \pm 3$$

$$|x| - 1 = 2 \pm 3$$

(+)

(-)

$$|x| - 1 = 5 \Rightarrow |x| = 5 \Rightarrow x = \pm 5$$

$$|x| - 1 = -5 \Rightarrow |x| = -4$$

$$|x| = 6$$

$$|x| = -4$$

Discarded

$$x = \pm 6$$

Ans

$$x \in \{-6, 6\}$$

$$|x| - 1 = -1$$

Discarded



Homework

① Revise full lecture..

JBK...JBK...

- TO *FUTURE IIT-IANS*



Thank – You

- *FUTURE IIT-IANS*

